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**Perova M.B., Perov E.V.**

*mperova@mail.ru, ev@perov.ru*

# ANALYSIS OF SHADOW OUTPUT ON THE ENTERPRISE

Shadow economic activity is an integral part of the formal economic activity. To some extent the shadow economic activity exist in every country in the world. When the scale of a shadow economy becomes very noticeable, it should be taken into account in economy management. The estimation of shadow economy scales is urgent not only at the level of a region and a country, but also at the level of an enterprise.

There are legitimate per se entities' economic activities which demand registration, which are not taken into consideration in the official accountancy and which, as a rule, cause decrease in assignments to the budget and to non-budget funds. Such entities' economic activities are here meant by *the shadow economy.*

Like of any other economic activities the factors of entities' economic activities are: maximum profit derivation and the mentality inherent to the society. All in all the populace is predisposed to work more in case of having opportunity to earn more. While the economic recession in the country takes place and decrease in standard of living of significant part of the population to a lower than living wage level takes place as well the satisfaction of primary needs (physiological by Maslow) becomes of a vital importance. When there is a lack of food stuff, the population is obliged to search for an additional income opportunity. This promotes the informal employment development and workers' greater predisposition to take part in shadow economic activities.

Excessive tax burden, redundant economic activities regulation and considerable scale of public sector in the economy – these ones are traditionally attributed to the basic reasons which cause the shadow activities growth. These reasons are common for countries with different economy types: market economy, transitional economics and development economics.

## 1. The shadow economy parameters revealing and estimation methods

While studying shadow economy researchers generally have one of the following purposes: a fundamental theoretical analysis, a statistical estimation, a social and economic policy optimization, perfection of law-enforcement activity, economic safety maintenance.

All the shadow economy revealing and estimation methods can be conditionally united into three groups: open-check methods, special economic-legal methods and statistical methods. For the purposes of the given research statistical methods are most applicable. Among them the following methods can be pointed out (Bekryashev, Belozyorov, 2000):

- *method of specific indicators* is connected with the use of the parameter which reflects an economic activities level, and received in the direct or indirect way;
- *method of soft modelling* (an determinants' estimation) is used for the relative volumes of shadow economy calculation through the distinguished set of factors, determining the shadow economy;
- *structural method* is based on the use of the information about shadow economy scales in various branches of production;
- *expert method* is based on the use of intuition and experience of the qualified specialists who define the degree of data verification, define interrelations and relations hardly described quantitatively;
- *mixed method* means usage of a latent variables method and a complex of various methods for the shadow economic activities level estimation. This method is based on the modeling, where the model considers a big number of factors which influence the shadow economy development. The shadow economy itself is considered as the latent variable which is not measured directly (a latent variables' method).

Within the specific indicators' method direct and indirect methods are being included.

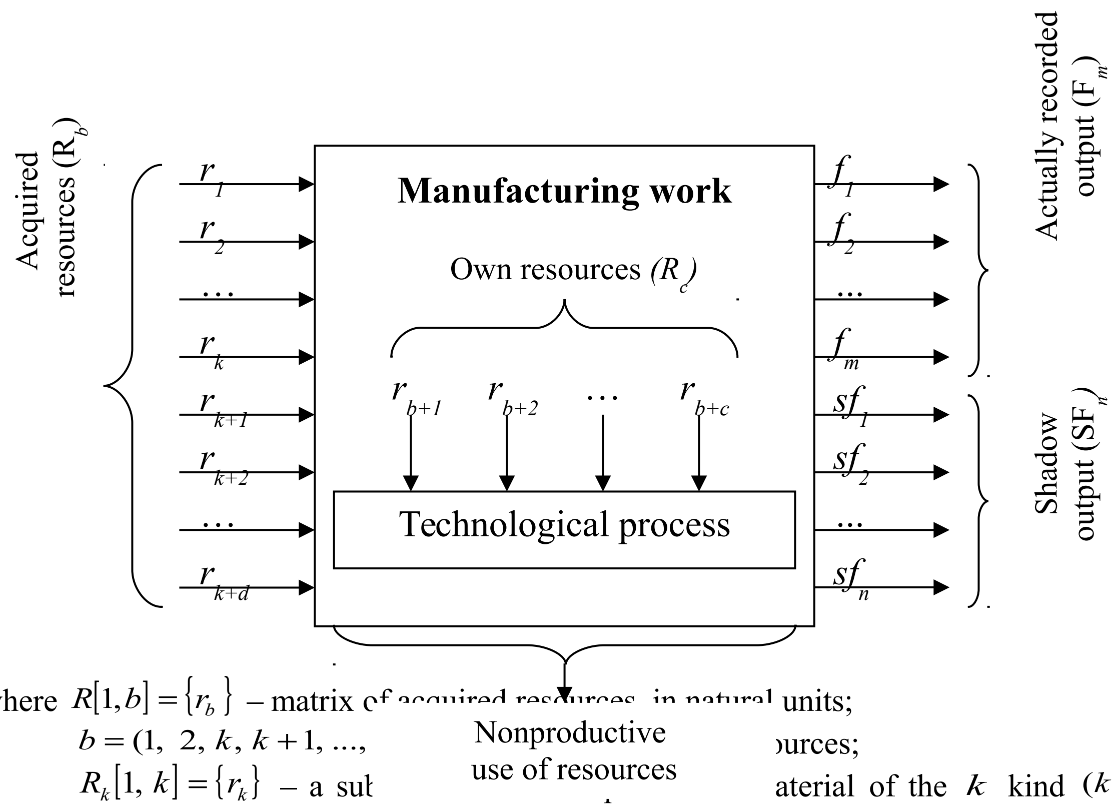
Direct methods assume the use of special inspections, the surveys for revealing divergences between incomes and expenditures and for the description of different aspects of shadow activity in groups of taxpayer (Mogensen et al, 1995; Pedersen, 2003).

Indirect methods are applied while having impossibility of researched parameters direct fixing. They are based on the official statistics information, the financial and tax authority data. Among the indirect methods some methods can be distinguished like: method of divergences, the Italian method (based on a employment parameter), a monetary method and their modifications (Giles, 1999a; Giles, 1999b; Thomas, 1992).

This or that method usage depends on the research aims and goals. The method of divergence is used in this research. It consists in usage of different methods to receive data from the same sources. The author's approach is implemented in three methods: commodity-normative and resource-normative methods of shadow output estimation, and resource-normative method of shadow economy estimation as well. The resource-normative method of shadow output estimation is proposed in this article.

## 2. Normative-resource method of shadow output calculation

The normative-resource method assumes the comparison between the resource quantity required for the commodity output production which is given in the economic reports and the real resource consumption. Consequently that total resource part which was not included in the actual reported output forming the shadow economy is sorted out. Schematically this process is represented in figure 1.




where $R[1,b]=\{r_b\}$ – matrix of acquired resources, in natural units;

$b=(1,\ 2,\ k,\ k+1,\ ...,$ … ources;

$R_k[1,\ k]=\{r_k\}$ – a sub… aterial of the $k$ kind $(k\in b)$, in natural units;

$r_1,\ r_2,\dots,\ r_k$ – expenditure of the $k$ acquired raw material, in natural units;

$r_{k+1},\ r_{k+2},\dots,\ r_{k+d}$ – expenditure of the $d$ acquired resource (except for raw material) like: the electric power and water for the technological purposes, heat, fuel and so forth, in natural units;

$R[1,\ b+c]=\{r_{b+c}\}$ – own resources matrix spent on output production, in natural units;

$r_{b+1},\ r_{b+2},\dots,\ r_{b+c}$ – expenses of $c$ own resource needed for the production of the given volume of output: labour, the basic output assets in the form of depreciation charges and so forth, in natural units;

$F[m,\ 1]=\{f_m\}$; $SF[n,\ 1]=\{sf_n\}$ – matrixes of actually recorded and shadow output produced from $j$ kinds of resources, in units of output. Shadow output is understood as produced output, but not recorded in the official reporting of the enterprise;

$j=(1,\ 2,\ ...,\ b,\ b+1,\ ...,\ b+c)$ – kinds of the acquired and own resources necessary for the output production ($b\in j,\ c\in j$);

$m=(1,\ 2,\ ...,\ \mu)$ – kinds of output being produced .

Fig. 1. The general approach to the shadow output estimation

Initially the matrix of normative expenditure of $j$ resources in real terms required for the production of a unit of $m$ output kinds is formed. With this basic normative specific values of resource expenditure are anticipatorily being amended with a glance to technological process changes relevant to the project.

$\ddot{R}^{н}[m, j] = \ddot{R}^{бн}[m, j] \cdot k[m, j]$,

each element of which is calculated as:

$\ddot{r}^{н}_{m\,j} = \ddot{r}^{бн}_{m\,j} \cdot k_{m\,j}$,

where $m = (1 \ldots \mu)$ – kinds of output being produced;

$j = (1 \ldots J)$ – kinds of the acquired and own resources necessary for the output production;

$\ddot{R}^{н}[m, j] = \{\ddot{r}^{н}_{m\,j}\}$ – matrix of a normative expenditure of *j* resource on *m* output unit production, in natural units / a unit of output;

$\ddot{R}^{бн}_{j}[m, j] = \{\ddot{r}^{бн}_{mj}\}$ – matrix of basic normative expenditure of *j* resources on *m* output unit production according to the project of object, in natural units / a unit of output;

$k[m, j] = \{k_{mj}\}$ – matrix of correcting coefficients of a normative *j* resources expenditure on the *m* output unit production, which takes into account the change of technological process relative to the project, in relative units ;

The matrix of actually expended resources on the quantum of output *m* is being formed on the basis of the current enterprise reporting data:

$R^{ф}[m, j] = \{r^{ф}_{mj}\}$.

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Normative resources expenditure for the actually produced output is calculated as multiplication of normative specific expenditure of the resource on actual volume of product output. The resources quantity which should be consumed according to the normative for actual volume of product output is calculated this way. The matrix of normative resource expenditure for the actually produced output *m* will be determined:

$R^{н}[m, j] = \ddot{R}^{н}[m, j] \cdot F^{ф}[m, 1]$

each element of which is calculated as:

$r^{н}_{mj} = \ddot{r}^{н}_{mj} \cdot f^{ф}_{m}$

where $R^{н}[m, j] = \{r^{н}_{mj}\}$ – matrix of normative resource expenditure for the actually produced output *m*, in natural units;

$\ddot{R}^{н}[m, j] = \{\ddot{r}^{н}_{mj}\}$ – matrix of normative resource expenditure on a unit of output *m*, in natural units / a unit of output;

$F^{ф}[m, 1] = \{f^{ф}_{m}\}$ – matrix of actually recorded output *m*, in units of output.

Elements of a matrix of normative *j* resources expenditure of the whole actually produced quantum of output $R^{н}[1, j]$, are calculated as the sum of resources expenditure for all *m* output kinds produced:

$$r_j^н = \sum_m r_{mj}^н ,$$

where $r_j^н$ – normative *j* resources expenditure of the whole actually produced output, in natural units;

$r_{mj}^н$ – normative *j* resource expenditure of the whole actually produced *m* output, in natural units.

Then the quantity of actually expended resources which exceeds normative requirement is estimated. This quantity could be expended for the shadow output production or could be lost for production because of the larcenies and negligence. For these terms we subtract normative resources expenditure from actually expended resources:

$$R^S[1,j] = R^Ф[1,j] - R^н[1,j];$$

$$r_j^S = r_j^ф - r_j^н ,$$

where $R^S[1,j] = \{r_j^S\}$ – matrix of resources, which could be expended for the shadow output production, in natural units;

$R^Ф[1,j] = \{r_j^ф\}$, $R^н[1,j] = \{r_j^н\}$ – matrixes of actual and normative resource expenditure on production of the whole actually produced quantity of output, in natural units.

It is necessary to find out how many units of output are possible to produce out of these $R^S$ resources. At constant structure of output the matrix of normative resource expenditure is calculated on one relative or conditional unit in natural units. The use of a relative or conditional unit depends on the enterprise being analyzed and a output kind.

*The relative unit of output* is used in case of several not interrelated output kinds' production, for example, the production of several sausage kinds at a meat processing and packing factory.

The elements of a matrix of normative resource expenditure on one relative unit $\ddot{R}^{нп}[1,j] = \{\ddot{r}_j^{нп}\}$ are calculated as arithmetic average of resource specific consumption $\ddot{r}_{mj}^н$ to production of output *m* kind, weighed on a quantity (or a share) of the given output kind:

$$\ddot{r}_j^{нп} = \frac{\sum_m \ddot{r}_{mj}^н f_m^ф}{\sum_m f_m^ф},$$

where $\ddot{r}_j^{нп}$ – the specific consumption of *j* resource on one relative unit, in natural units / a relative unit of output;

$\ddot{r}_{mj}^н$ – the specific consumption of *j* resource on output *m* kind, in natural units / a unit of output;

$f_m^ф$ – actually produced quantity of output *m* kind, in units of output.

Dividing the shadow resource matrix by matrix of expenditure of the resource on one relative output unit, gives relative units quantity which can be produced out of every shadow resource kind:

$$SF^{n}[1,j] = R^{S}[1,j] : \ddot{R}^{нп}[1,j];$$

$$sf_j^n = \frac{r_j^S}{\ddot{r}_j^{нп}},$$

where $SF^{n}[1, j] = \{sf_j^n\}$ – matrix of relative output units, which can be produced out of every shadow resource kind, in natural units;

$R^{S}[1, j] = \{r_j^S\}$ – matrix of resource, which could be spent for shadow output production, in natural units;

$\ddot{R}^{нп}[1,j] = \{\ddot{r}_j^{нп}\}$ – normative matrix of the resource expenditure on one relative output unit, in natural units / a relative unit of output.

In consideration of the shadow output volume as the probabilistic rate, a maximum quantity of shadow output is accepted as equal to the minimal value of relative output units. This is the output which can be produced out of *u* resources:

$$sf_{max}^n = [sf_u^n]_{min}, \qquad u \in j$$

where $sf_{max}^n$ – maximal quantity of relative shadow output, in relative units of output;

$[sf_u^n]_{min}$ – minimal quantity of output, which can be produced from *u* resources, in relative units of output;

*u* – kinds of resources, which cannot be acquired from the outside without a record in the official reporting, for example the electric power or water, in natural units.

The minimal quantum of shadow output is equal to minimal value of relative output which is possible to be produced out of all the *j* resources:

$$sf_{min}^n = [sf_j^n]_{min},$$

where $sf_{min}^n$ – minimal quantity of shadow output, in relative units of output;

$[sf_j^{n(y)}]_{min}$ – minimal quantity of the relative output, which can be produced out of *j* resources, in relative units of output.

In case of lack of additional information the quantum of shadow output in relative units is calculated as a mathematical expectation:

$$M(sf^n) = [sf_{max}^n,\ sf_{min}^n],$$

where $M(sf^n)$ – mathematical expectation of shadow output volume, in relative units;

$sf_{max}^n$, $sf_{min}^n$ – maximal and minimal shadow output quantity, in output relative units.

The mathematical expectation of shadow output cost in enterprise release prices is calculated by the price of relative output unit:

$$M(S) = M(sf^n) \cdot p^n;$$

$$p^n = \frac{\sum_m p_m f_m^ф}{\sum_m f_m^ф},$$

where $M(S)$ – mathematical expectation of shadow output cost in release prices of the enterprise, in roubles;

$M(sf^n)$ – mathematical expectation of shadow output volume in relative units;

$p^n$ – release price of output relative unit, in roubles / output relative unit;

$p_m$ – release price of *m* kind output, in roubles / output unit;

$f_m^ф$ – actual quantity of output *m* kind, in output units.

*Conditional output unit* is used on enterprises where the dissimilar but interrelated output is produced. For example, the main output kind at cattle industrial fattening enterprises is meat; in edition, incidental output is sold such as manure or peat-manure compost.

Considering as known the collateral output quantity on a unit of the basic output the resource expenditure is being reduced to one conditional unit:

$$\ddot{R}^{ну}[1, j] = \{\ddot{r}_j^{ну}\};$$

$$\ddot{r}_j^{ну} = \ddot{r}_{осн\,j}^{н} + \ddot{r}_{поб\,j}^{н} \cdot \ddot{f}_{поб/осн}^{бн},$$

where $\ddot{R}^{ну}[1, j] = \{\ddot{r}_j^{ну}\}$ – matrix of the specific normative of expenditure of the *j* resource on one conditional unit, in natural units / output conditional unit;

$\ddot{r}_{осн\,j}^{н}$, $\ddot{r}_{поб\,j}^{н}$ – the specific normative of *j* resource expenditure on production of basic and collateral output, in natural units;

$\ddot{f}_{поб/осн}^{бн}$ – normative production of collateral output per unit of basic output, in natural units / natural units.

The quantity of output conditional units, which can be produce out of each shadow resource kind, is equal to:

$$SF^y[1,j] = R^S[1,j] : \ddot{R}^{ну}[1,j];$$

$$sf_j^y = \frac{r_j^S}{\ddot{r}_j^{ну}},$$

where $SF^y[1,j] = \{sf_j^y\}$ – matrix of output conditional units, which can be produce out of each shadow resources kind, in natural units;

$R^S[1, j] = \{r_j^S\}$ – matrix of resource quantity, which could be spend on production of shadow output, in natural units;

$\ddot{R}^{ну}[1,j] = \{\ddot{r}_j^{ну}\}$ – normative matrix of resource expenditure on one output conditional unit, in natural units / output conditional unit.

Maximal quantity of conditional shadow output, which can be produced out of *u* resources:

$$sf_{max}^y = [sf_u^y]_{min}, \qquad u \in j$$

where $sf_{max}^y$ – maximal quantity of shadow output, output conditional units;

$\left[sf_u^y\right]_{min}$ – minimal quantity of output conditional units, which can be produced out $u$ resources, in output conditional units;

$u$ –kinds of resources, which cannot be acquired from the outside without a record in the official reporting, in natural units.

The minimal volume of shadow output, which can be produced out all $j$ resources:

$$sf_{min}^y = \left[sf_j^y\right]_{min},$$

where $sf_{min}^y$ – minimal quantity of shadow output, in output conditional units;

$\left[sf_j^y\right]_{min}$ – minimal quantity of conditional output, which can be produced out all $j$ resources, in output conditional units.

The shadow output volume in conditional units is calculated as a mathematical expectation:

$$M(sf^y) = \left[sf_{max}^y,\ sf_{min}^y\right],$$

where $M(sf^y)$ – mathematical expectation of the shadow output volume in conditional units;

$sf_{max}^y$, $sf_{min}^y$ – maximal and minimal shadow output quantity, in output conditional units.

The mathematical expectation of shadow output cost in enterprise release prices is calculated by the price of output conditional unit:

$$M(S) = M(sf^y) \cdot p^y;$$

$$p^y = p_{осн} + p_{поб} \cdot \ddot{f}_{поб/осн}^{бн},$$

where $M(S)$ – mathematical expectation of shadow output cost in enterprise release prices, in roubles;

$M(sf^y)$ – mathematical expectation of the shadow output volume, in conditional units,

$p^y$ – release price of output conditional unit, in roubles / output conditional unit;

$p_{осн}$, $p_{поб}$ – release price of basic and collateral output, in roubles / output unit;

$\ddot{f}_{поб/осн}^{бн}$ – normative production of collateral output per a basic output unit, in natural units / natural units.

*The shadow output level* in relational expression in the total amount of produced output is the following:

$$s = \frac{M(S)}{F + M(S)},$$

where $s$ – shadow output level in relational expression in total amount of output, in a share;

$M(S)$ – mathematical expectation of shadow output cost in release prices of the enterprise, in roubles;

$F$ – recorded commodity output of the enterprise during the period being analyzed, in roubles.

## 4. Concluding remarks

The calculations executed with the given method on a cattle fattening complex have allowed to reveal the shadow output scale at the rate of 18 tons of the conditional output including meat in live weight and peat-manure compost. This output cost is 46 thousand euro in release prices.

Remarkable labor-output ratio of the correction of normative resource expenditure on each output kind is attributed to this method's shortcomings. However such a calculation for the given output production program is once done and then can be used till the output production program changes. In edition remarkable labor-output ratio is compensated by the precision of shadow output estimation at an enterprise.

The revelation of shadow output at an enterprise is of interest for enterprise owners as far as enterprise managers quite often manipulate the production and investment processes for self-profit. This method can be useful to audit firms and tax authority as well as to state and municipal authorities.

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